Fundamental Limitations of Small Antennas: Validation of Wheeler's Formulas

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Abstract

About sixty years ago, Harold A Wheeler published a paper entitled, "Fundamental Limitations of Small Antennas." Wheeler presented formulas that related the antenna radiation Q to the physical volume of the antenna. This paper (1) briefly reviews the history of the fundamental limitations of small antennas, (2) presents Wheeler's formulas in a revised form, and (3) uses the *WIPL-D* computer code to validate the Wheeler formulas. The goal is to demonstrate that the Wheeler paper did, indeed, address the fundamental limitations of electrically small antennas, and that it is very useful in the understanding and quantification of these limitations.

Keywords: Antenna theory; antenna measurements; small antennas; Q factor; Wheeler formulas

1. Introduction

In 1947, Wheeler [1] published the paper "Fundamental Limitations of Small Antennas." In the following year, Chu [2] published the paper "Physical Limitations of Omni-Directional Antennas." In later publications [3-10], Wheeler expanded and explained his concepts for small antennas. References [11-17] credit Chu as the major contributor to the understanding of the fundamental limitations of small antennas. The fact is that both Wheeler and Chu made major contributions to the theory of small antennas.

Wheeler [1] introduced the concept of lumped-element electrically small antennas, and defined the fundamental relationship between the radiation Q of lumped-element electrically small antennas and their physical (occupied) volume:

$$Q_{Wheeler} = \frac{9}{2}\frac{V_{RS}}{V_E}, \tag{1}$$

where V_{RS} is the volume of the radian sphere (with a radius of $\lambda/2\pi$), and V_E is the effective volume, which is directly related to the physical (occupied) volume of electrically small antennas.

Chu [2] defined the theoretical lower bound for the radiation Q of electrically small antennas as

$$Q_{Chu} = \frac{1}{\left(\frac{2\pi r}{\lambda}\right)^3} = \frac{V_{RS}}{V}, \tag{2}$$

where V is equal to the volume of the sphere with a diameter, $2r$ (where r is the radius of the sphere), equal to the greatest dimension of the electrically small antenna. This is the antenna's spherical boundary. Note: The Chu lower-bound antenna has no stored energy (reactive power) inside the spherical boundary. It can only be realized in theory.

The radiation Q, or Q of a resonant antenna, is defined as the ratio of 2π times the energy stored in the fields excited by the antenna to the energy radiated and dissipated per cycle [18].

The goal of this paper is to present Wheeler's formulas in a format more consistent with current practice, and to demonstrate the validity of his formulas by means of computer simulation. As is typical of Wheeler, his work on the fundamental limitations of small antennas is helpful to theoreticians and practitioners. His formulas have been the basis for many practical designs of small antennas, such as the largest electrically small antennas, the VLF transmitters at Cutler, ME, and at Northwest Cape, Australia [9].

Wheeler [1] defined a small antenna as follows: "The small antenna to be considered is one whose maximum dimension is less than the 'radianlength.' The radianlength is $1/2\pi$ wavelength." In this paper, the electrically small antenna is defined as one the greatest dimension of which is less than one-tenth wavelength ($2r/\lambda < 1/10$). For this range of antenna size, there is universal agreement with the Chu lower bound for antenna radiation Q. This also defines a conservative range for the validity of the lumped-element antenna assumption.

The history of this subject has been researched, and it is the author's belief that Harold A. Wheeler was, indeed, the first one to directly relate antenna radiation Q to antenna volume.

Apart from the lumped-element capacitor and inductor antennas, there are other types of electrically small antennas. These are the combination of the capacitor and inductor lumped-element antennas, and self-resonant distributed-element antennas [19]. This paper concentrates on the basic capacitor and inductor lumped-element antennas.

2. Wheeler's Formula for the Inductor Antenna

Wheeler's fundamental concept for electrically small antennas is presented in Figure 1, which is reproduced from [1]. Wheeler states, "An antenna within this limit of size can be made to behave essentially as lumped capacitance or inductance, so this property is assumed." The capacitor (electric) antenna or the inductor (magnetic) antenna has an associated radiation resistance. The radiation Q of the antenna is equal to the lumped-element reactance divided by the radiation resistance. Wheeler [1] presents formulas for the radiation power factor, p, which is equal to $1/Q$. The inductor (loop) antenna is considered first.

Wheeler's formula [1] for the Q of the loop antenna is presented in Equation (3):

$$Q_{Wheeler(Inductor)} = 6\pi \frac{\left(\frac{\lambda}{2\pi}\right)^3}{\pi a^2 b} \frac{1}{k_b} = \frac{9}{2}\frac{V_{RS}}{V_E}, \tag{3}$$

where a is the loop radius; b is the loop axial length; $k_b = k_{SL} k_{FL}$ is the effective volume factor for the inductor antenna; k_{SL} is the shape factor for the inductor antenna, with $k_{SL} \approx 1 + 0.9\frac{a}{b}$ (if $b < a$, the factor is somewhat less than this value); k_{FL} is the fill factor for the inductor antenna, with

$$k_{FL} \approx \frac{1}{1 - \frac{1}{k_{SL}}\left(\frac{\mu_r - 1}{\mu_r}\right)} \text{ for } \frac{b}{a} > 2,$$

where μ_r is the relative permeability of the filling (core) material; and λ is the free-space wavelength.

For $\mu_r = 1$, the Wheeler formula for the Q of the air-core inductor antenna is given by

$$Q_{Wheeler(Inductor)} \approx \frac{9}{2} \frac{V_{RS}}{\pi a^2 b\left(1 + 0.9\frac{a}{b}\right)}. \tag{4}$$

It is noted that the Q is directly related to the cylindrical volume occupied, $\pi a^2 b$. This is a fundamental relationship: the radiation Q of an electrically small antenna is fundamentally limited by its physical volume.

The current development of electromagnetic computer codes is such that a computer simulation of the Wheeler inductor antenna is routine. The *WIPL-D* code [20, 21] was used for this purpose.

3. Validation of Wheeler's Formula for the Inductor Antenna

The *WIPL-D* model for the simulation of the Wheeler inductor antenna is presented in Figure 2. Wheeler states in [1] that "The inductor (loop antenna) is assumed to act as a current sheet." The perfect electrically conducting plates of the *WIPL-D* model provided a good simulation of the Wheeler current-sheet configuration. A wire was required in the model to excite the loop. The wire diameter was set equal to $b/2$.

A comparison of the *WIPL-D* results with the Wheeler formula is presented in Figure 3. It was observed that good agreement was achieved, validating Wheeler's formula for the inductor antenna.

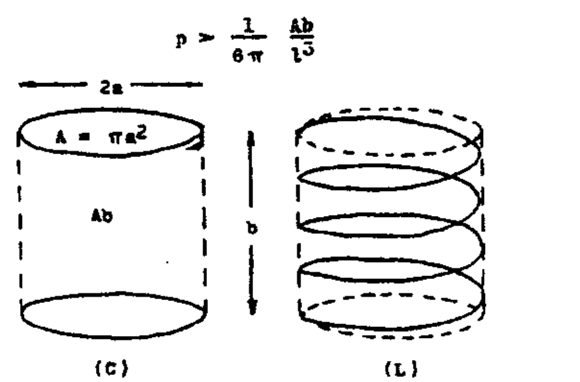

Fig. 1—Capacitor (*C*) and inductor (*L*) occupying equal cylindrical volumes.

Figure 1. The original figure appearing in Wheeler's 1947 paper [1]: $p = 1/Q$, $l = \lambda/2\pi$.

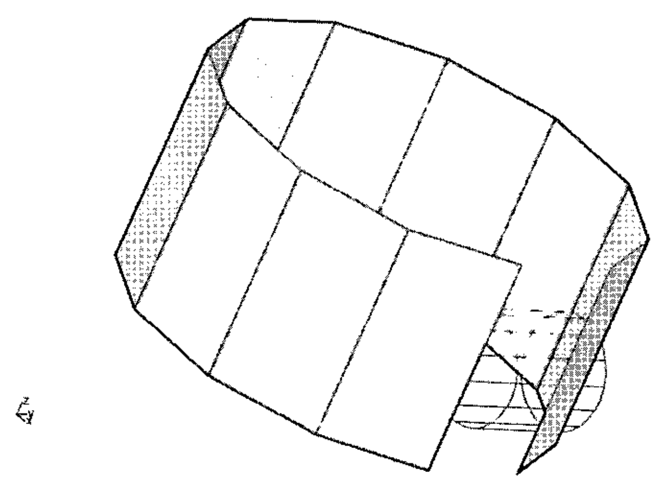

Figure 2. The *WIPL-D* model for simulation of Wheeler's inductor antenna.

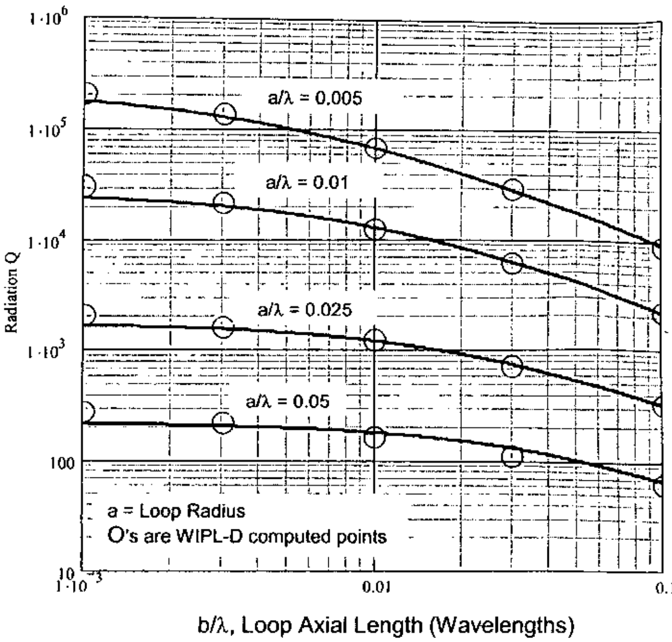

Figure 3. Wheeler's formula for the radiation Q of the inductor (loop) antenna, Equation (4), and the *WIPL-D* simulation results. The *WIPL-D* points should be in agreement with the closest trace. The point at $a/\lambda = 0.5$ and $b/\lambda = 0.1$ corresponds to a geometry that is slightly beyond the assumed electrically small boundary.

For a single-turn loop with a radius that is less than one-fortieth of a wavelength, the loop antenna is essentially an inductance-only antenna, and the Q is given by

$$Q = \frac{X}{R}, \tag{5}$$

were $Z = R + jX$ and X is the net reactance of the loop antenna.

This is not true for the electric (capacitor) antenna, and the assumption that it is valid can lead to erroneous results, as discussed below.

4. The Chu Antenna and Wheeler's Implementation

In December, 1948, Lan Jen Chu published the paper "Physical Limitations of Omni-Directional Antennas." His primary goal was to quantify the super-gain limitations of omnidirectional antennas. In the process, he derived the equation for the fundamental theoretical minimum Q for electrically small antennas. His solution was in terms of spherical waves. He was able to associate the spherical-wave solution with an equivalent lumped-element circuit. The simplest of all the equivalent circuits corresponded to that of an infinitesimally small dipole. This simple circuit can be used to derive the equation for the fundamental theoretical lower bound for the Q of an electrically small antenna. (Reference [16] provides an excellent overview and description of Chu's work.)

The Chu antenna has the following attributes:

1. It is enclosed within a circle with a diameter that is equal to the greatest dimension of the antenna. (In this paper, r is the radius of the Chu sphere.)

2. There is no stored energy or reactive power inside the sphere.

Figure 4 is Figure 3 of Chu's original paper. It can be used to simply derive Chu's formula. The input impedance is given by

$$Z = \frac{-j}{\omega C} + \frac{j\omega L}{1+(\omega L)^2} + \frac{(\omega L)^2}{1+(\omega L)^2},$$

$$Q_{Chu} = \frac{1+(\omega L)^2}{\omega C(\omega L)^2} = \frac{1+\left(\omega \frac{r}{c}\right)^2}{\omega \frac{r}{c}\left(\omega \frac{r}{c}\right)^2} = \frac{1+\left(\frac{2\pi r}{\lambda}\right)^2}{\left(\frac{2\pi r}{\lambda}\right)^3},$$

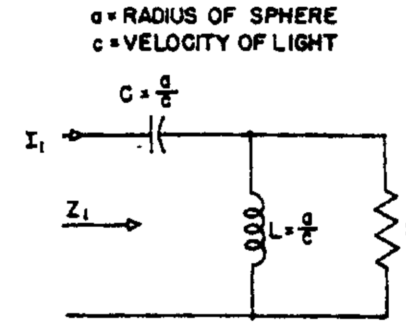

FIG. 3. Equivalent circuit of electric dipole.

Figure 4. The original Chu figure [2] for the equivalent circuit for an infinitesimally small electric dipole.

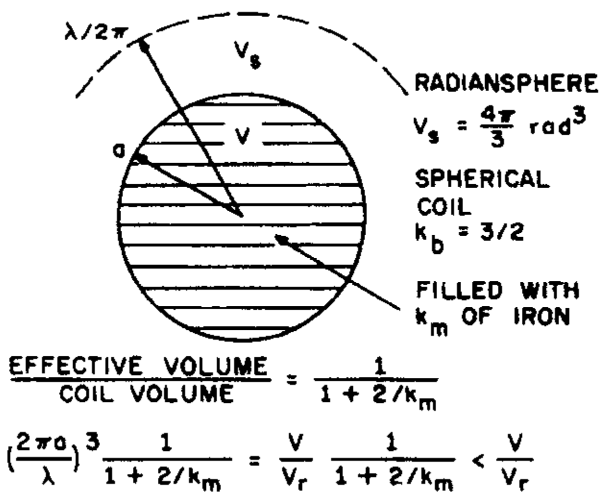

FIG. 6-5 The spherical coil with a magnetic core.

Figure 5. Wheeler's Chu antenna: k_m is the relative permeability of the inductor core, which is infinite. (This is Figure 5 in [7] and Figure 6-5 in [8]. In the third equation, $1/[1+2/k_m]$ should be multiplied by $9/2$, and in the bottom equation, $V_r = V_s$. The bottom equation defines the upper bound for the radiation power factor, which, when inverted, is the lower bound for the radiation Q, and which is in agreement with the Chu lower bound.)

$$Q_{Chu} \approx \frac{(\lambda/2\pi)^3}{r^3} = \frac{V_{RS}}{V}, \text{ for } 2r < \frac{\lambda}{10}.$$

In [3] and [8], Wheeler described a physical antenna that was indeed a Chu antenna: it had a physical spherical volume and there was no stored energy within the volume. Wheeler's Chu antenna is shown in Figure 5, which is shown as originally published [8]. It is a spherical coil with an infinite-permeability core. (Wheeler, in [6], pointed out that it was Maxwell who first described the spherical coil in his monumental treatise.) Wheeler, [8], wrote:

> There is one theoretical case of a small antenna which has the greatest radiation PF [smallest radiation Q] obtainable within a spherical volume. Figure 6-5 [Figure 4] shows such an antenna and its relation to the radiansphere (V_s).[1,2] It is a spherical coil with a perfect magnetic core. The effective volume of an empty spherical coil has a shape factor $3/2$. Filling with a perfect magnetic core ($k_m = \infty$) multiplies the effective volume by 3.
>
> $$p_m = \frac{2}{9}\frac{(3)(3/2)V}{V_s} = \frac{V}{V_s} = \left(\frac{2\pi a}{\lambda}\right)^3 \qquad (6\text{-}7)$$
>
> This is indicated in the shaded sphere a.
>
> This idealized case depicts the physical meaning of the radiation PF [Q/] that cannot be exceeded [made lower]. Outside the sphere occupied by the antenna, there is stored energy or reactive power that conceptually fills the radiansphere, but there is none inside the antenna sphere. The reactive power density, which is dominant in the radiation within the radiansphere, is related to the real power density, which is dominant in the radiation outside.
>
> In a rigorous description of the electromagnetic field from a small dipole of either kind, the radiation of power in the far field is accompanied by stored energy which is located mostly in the near field (within the radiansphere). The small spherical inductor in Fig. 6-5 [Figure 5] is conceptually filled with perfect magnetic material, so there is no stored energy inside the sphere. This removes the *avoidable* stored energy, leaving only the *unavoidable* amount outside the inductor but mostly inside the radiansphere. This unavoidable stored energy is what imposed a fundamental limitation on the obtainable radiation PF [Q].

Note: The comments within the [ ] brackets were inserted by the author. They are used primarily to convert the power factor (PF) concept to that of Q.

The Q for Wheeler's Chu antenna is given by

$$Q_{Wheeler} = \frac{V_{RS}}{V} = Q_{Chu}, \qquad (6)$$

where V is the volume of the spherical coil. The Q for the spherical coil with an air core is given by

$$Q_{Wheeler-Air\ Spherical\ Coil} = 3\frac{V_{RS}}{V}. \qquad (7)$$

A convenient figure-of-merit for electrically small antennas is the Q ratio, QR. It is defined as

$$QR = \frac{Q}{Q_{Chu}}, \qquad (8a)$$

where Q is the radiation Q of the electrically small antenna under consideration, and

$$Q_{Chu} = \frac{V_{RS}}{V_{Chu}}, \qquad (8b)$$

where V_{Chu} is the volume of a sphere with a diameter equal to the largest dimension of the small antenna. The Q ratio is the ratio of the total stored energy to the external stored energy.

Equation (7) shows that for the air-core spherical coil, $QR = 3$: the total stored energy, internal plus external, is three times the external energy.

5. Wheeler's Formula for the Capacitor Antenna

The Wheeler capacitor antenna, Figure 1, is a hypothetical antenna: it is not possible to have a capacitance-only antenna. A practical implementation of Wheeler's capacitor antenna requires a wire excitation of the two discs, which results in a disc-dipole antenna. The wire introduces inductance and, consequently, it is not possible to simply simulate the capacitance-only antenna, as was done for the inductor (loop) antenna, which is primarily an inductance-only antenna.

The Wheeler formula [1] for the capacitor antenna is given by

$$Q_{Wheeler(Capacitor)} = 6\pi\frac{\left(\frac{\lambda}{2\pi}\right)^3}{\pi a^2 b}\frac{1}{k_a} = \frac{9}{2}\frac{V_{RS}}{V_E}, \qquad (9)$$

where a is the disc radius; b is the distance between the discs, dipole length; $k_a = k_{SC}k_{FC}$ is the effective volume factor for the capacitor antenna; k_{SC} is the shape factor for the capacitor antenna, with $k_{SC} \approx 1$ for $b/a \ll 1$ and $k_{SC} \approx (4/\pi)(b/a)$ for $b/a \gg 1$; k_{FC} is the fill factor for the capacitor antenna, with $k_{FC} \approx \frac{1}{1+\frac{\varepsilon_{r-1}}{k_{SC}}}$ for $b/a < 2$; ε_r is the relative permittivity of the fill (core) material; and λ is the free-space wavelength.

In a later internal memorandum [10], Wheeler presented the following formula for k_{SC}:

$$k_{SC} = 1 + \frac{4}{\pi}\frac{b}{a}. \qquad (10)$$

This equation is validated in Section 6, below.

The Q for the air-core disc dipole antenna is given by

$$Q_{Wheeler(Capacitor)} \approx \frac{9}{2} \frac{V_{RS}}{\pi a^2 b \left(1 + \frac{4}{\pi} \frac{b}{a}\right)}. \tag{11}$$

6. Validation of Wheeler's Formula for the Capacitor Antenna

The electric antenna is fundamentally different from the magnetic antenna in that the net reactance consists of two components, a capacitive component and an inductive component. The Q of an electrically small antenna is not equal to the net reactance divided by the resistance. If the impedance is given by $Z = R + jX$ and $X = -X_C + X_L$, then

$$Q = \frac{X_C}{R} \neq \frac{X}{R}. \tag{12}$$

This relationship makes the computation of the Q for the electric antenna more difficult than that of the Q for the magnetic antenna. A procedure can be implemented for using computer codes to calculate the Q. It is described below for the *WIPL-D* code.

The *WIPL-D* code and the following formula [22] are used to determine the Q of the disc dipole:

$$Q = \frac{f}{\Delta f} \frac{\Delta X}{2R} + \frac{|X|}{2R}, \tag{13}$$

where f is the geometric-mean frequency, Δf is the frequency interval, ΔX is the absolute value of the reactance change through the frequency interval, R is the resistance, and $|X|$ is the absolute value of the reactance at the geometric-mean frequency. To determine the Q, computations are made at two frequencies near resonance to quantify $\Delta X / \Delta f$.

The *WIPL-D* simulation of the disc dipole antenna is shown in Figure 6. The results of the simulation are shown in Figure 7, along with the plots of the Wheeler disc-dipole formula, Equation (11). Very good agreement was observed between the Wheeler disc-dipole formula and the *WIPL-D* computed points. This provides a validation of the Wheeler formula for the capacitor antenna.

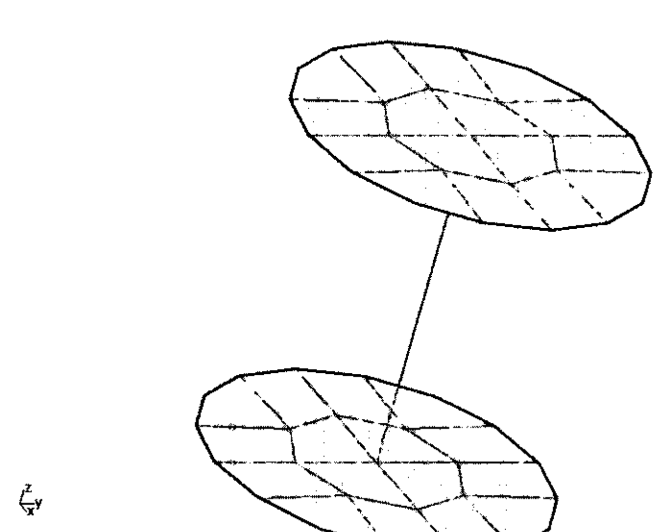

Figure 6. The *WIPL-D* simulation for the disc-dipole antenna.

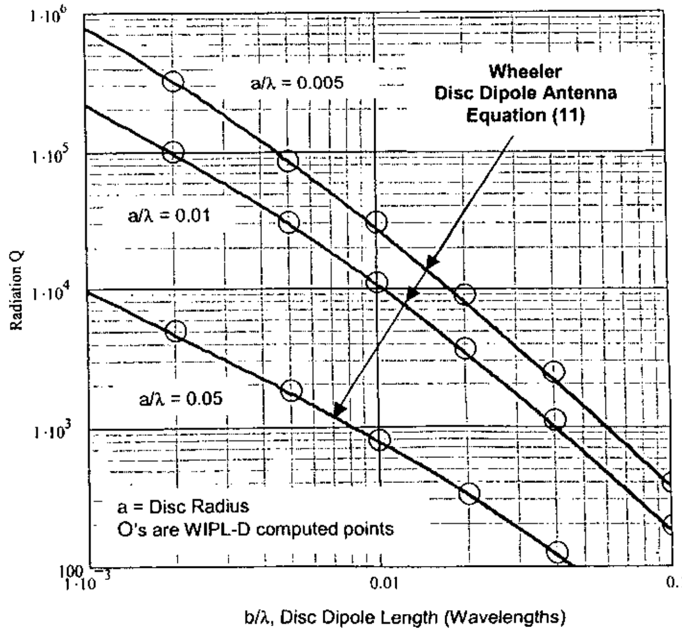

Figure 7. A comparison between the predictions for the disc-dipole antenna's Q based on Equation (11) and the Q computed with *WIPL-D*, for selected points.

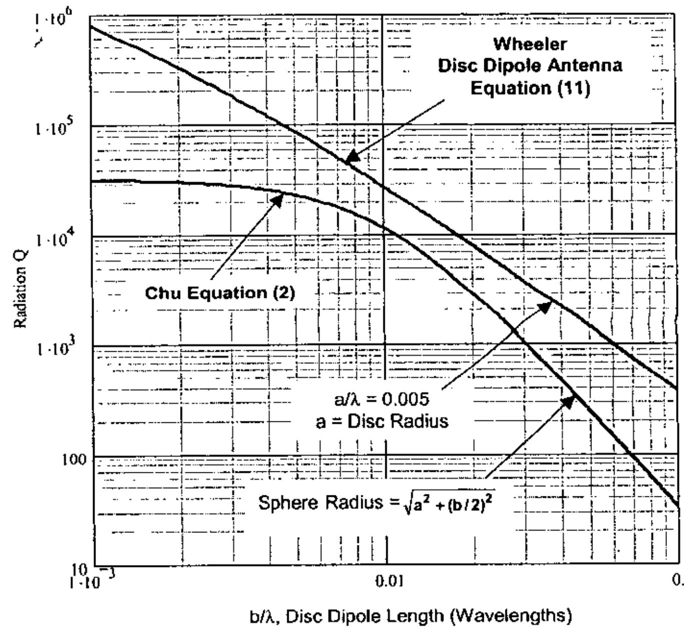

Figure 8. A comparison of the Wheeler lower bound for electric antennas enclosed within a cylinder and the Chu lower bound.

7. Lower Bound for Cylindrical Electric Antennas

Wheeler described one antenna that achieved the Chu lower bound. For the spherical-coil antenna, he showed that the Chu limit was achieved when the relative permeability approached infinity. At low frequencies, very large values for μ_r are available.

For the case of the electric antenna, the situation is much different. For this antenna, there is no means for reducing the internal stored energy: increasing the relative dielectric constant increases the Q of the antenna. In this case, a lower bound is defined by the Wheeler capacitor formula, Equation (11). This lower bound is defined for any electric antenna that can be enclosed within a cylindrical volume. A comparison of the Wheeler lower bound for this class of electric antennas and the Chu lower limit is presented in Figure 8. (It is noted that a slightly lower Q – less than 5% – can be achieved if short skirts are added around the rim of the discs. This difference is not significant, so the simple disc is used as representative of the lower bound for this class of electric antennas.)

The key point here is that the Chu lower bound is too low for an electric antenna.

8. Cylindrical Dipole

The cylindrical-dipole antenna has been investigated extensively, and a formula for the Q of a short cylindrical-dipole antenna has been published [14]:

$$z \approx 20k^2h^2 - j\frac{120\left[\ln(h/a)-1\right]}{\tan(kh)},$$
$$Q \approx \frac{6\left[\ln(h/a)-1\right]}{k^2h^2\tan(kh)}, \tag{14}$$

where h is the half-length ($=b/2$), a is the radius of the cylinder, and $k = 2\pi/\lambda$. Equation (14) assumes that the Q is equal to the net reactance divided by the resistance. This, as discussed above, is not normally a valid assumption.

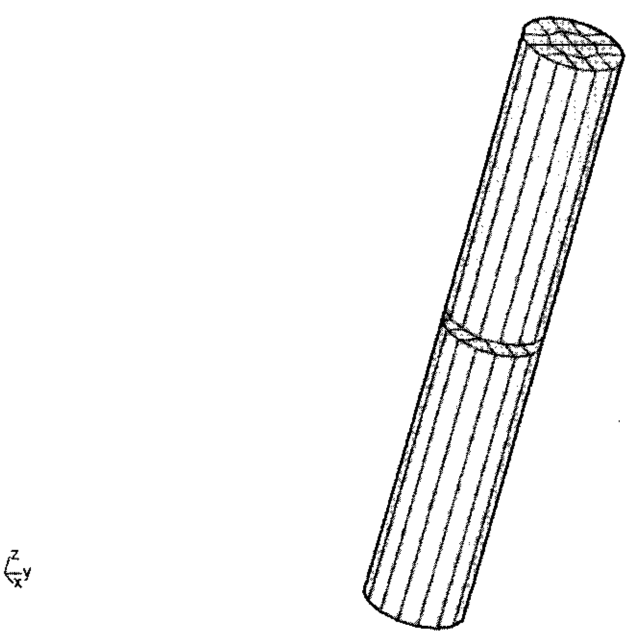

Figure 9. The *WIPL-D* simulation for the cylindrical dipole antenna.

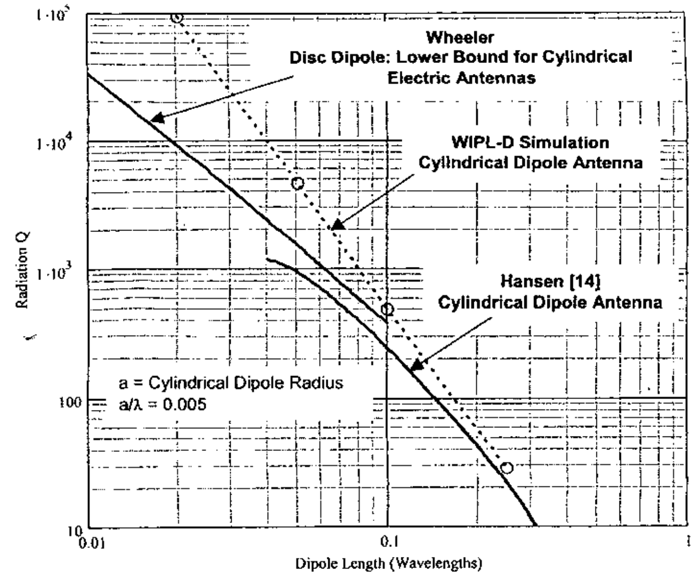

Figure 10. The *WIPL-D* computer simulation results and those from the Hansen formula for the cylindrical-dipole antenna, compared with the lower bound for electric antennas.

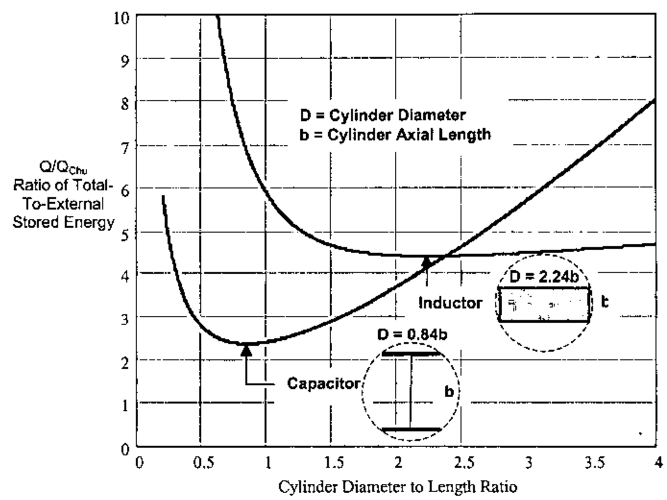

Figure 11. The cylinder shape (D/b ratio) for minimum Q for an air-core lumped-element antenna. The antenna's greatest dimension was 0.1λ to fit within a sphere of radius 0.05λ.

The *WIPL-D* computer model used to simulate the cylindrical dipole antenna is shown in Figure 9, and the results are shown in Figure 10. The computer simulation results showed that for dipole lengths less than 0.1 wavelength, the Q for the cylindrical-dipole antenna was higher than the Q for the disc-dipole antenna, which is consistent with Equation (11) defining the lower bound. Also included in Figure 10 is the formula from reference [14] for the cylindrical dipole.

9. Lumped Element Antennas: Optimum Shape to Fit Within a Sphere

The Wheeler formulas and the Chu formula can be used to determine the optimum shape factor (the D/b ratio, where $D=2a$

is the cylinder diameter) for the lumped-element antennas with air cores that provide the lowest Q, and that fit within a given sphere. For these antennas, the Q ratios for the magnetic and electric antennas are defined by the author, and are given by

$$QR_{Magnetic} = \frac{Q_{Wheeler(Magnetic)}}{Q_{Chu}} = \frac{9}{2}\frac{V}{V_E} = 6\frac{\left[1+\left(\frac{b}{2a}\right)^2\right]^{3/2}}{\frac{b}{a}\left(1+0.9\frac{a}{b}\right)}, \quad (15)$$

$$QR_{Electric} = \frac{Q_{Wheeler(Electric)}}{Q_{Chu}} = 6\frac{\left[1+\left(\frac{b}{2a}\right)^2\right]^{3/2}}{\frac{b}{a}\left(1+\frac{4}{\pi}\frac{b}{a}\right)}.$$

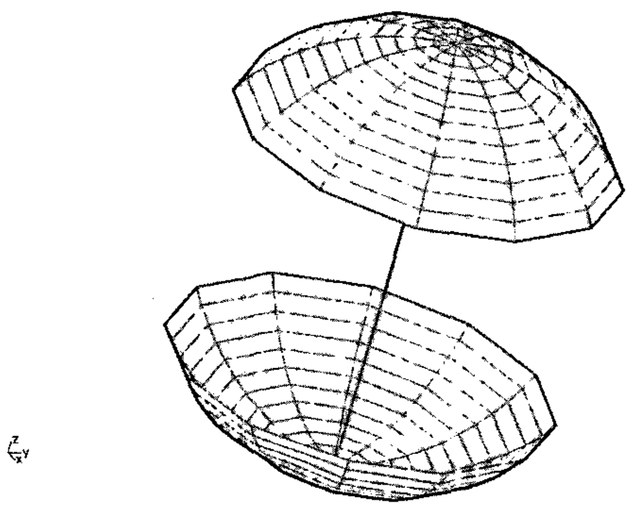

Figure 12. The *WIPL-D* simulation for a spherical-cap dipole antenna.

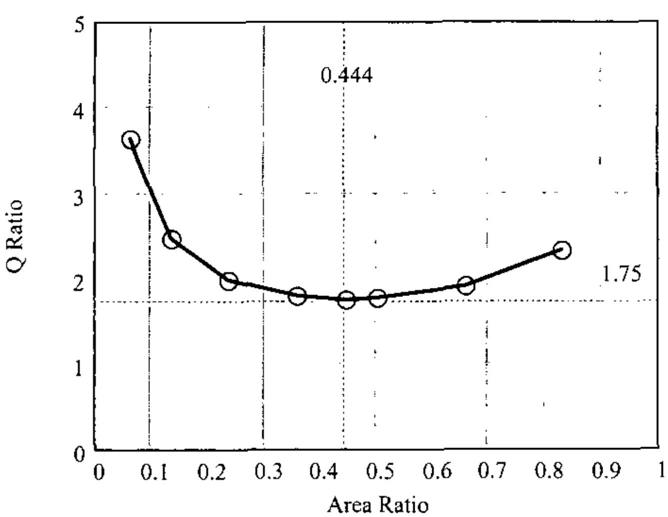

Figure 13. The Q ratio as a function of the ratio of the cap area to the sphere area for the spherical-cap dipole antenna. The sphere diameter for the *WIPL-D* model was 0.05λ.

Table 1. The Q ratio for basic antennas.

Antenna Type	Q Ratio $= Q/Q_{Chu}$ Ratio of Total to Externally Stored Energy
Spherical inductor, $\mu_r = \infty$	1.0
Spherical inductor, $\mu_r = 1$	3.0
Cylindrical inductor, $\mu_r = 1$, $D/b = 2.24$	4.4
Disc dipole, $D/b = 0.84$	2.4
Spherical-cap dipole	1.75

It can be determined that $QR_{Magnetic}$ is a minimum when $D = 2.24b$, and that $QR_{Electric}$ is a minimum when $D = 0.84b$. Plots of QR versus D/b are presented in Figure 10. It was noted that for the optimum configurations, $QR = 4.4$ and $D = 2.24b$ for the inductor antenna, and $QR = 2.4$ and $D = 0.84b$ for the capacitor antenna.

In [9], Wheeler pointed out that for a capacitor antenna, the "best simple utilization of a spherical volume is achieved by spherical caps covering about one half the surface" of the sphere. The *WIPL-D* code was used to determine the characteristics of the spherical-cap dipole antenna. The model is shown in Figure 12. As shown in Figure 13, the lowest radiation Q was achieved when the area was 0.444 of the spherical area, and then $QR = 1.75$. This is believed to be the lower bound for electric antennas.

Table 1 presents a summary of the Q ratios for the antennas considered in this paper.

10. Summary and Conclusions

This paper revisited the problem of electrically small antennas and the quantification of the fundamental limitations of such antennas. It highlighted the work of Harold A. Wheeler, which is presented in references [1, 3-10].

An often-cited reference in papers dealing with electrically small antennas is Chu's paper, [2]. Chu basically described the fundamental limitations of a hypothetical antenna. The Chu antenna has a spherical physical volume with a diameter that is equal to the greatest dimension of the antenna, and which has no stored energy (reactive power) inside the sphere. The Chu formula serves as a convenient point of reference for the comparison of alternative electrically small antennas.

This paper presented Wheeler's formulas in a format more consistent with current practice. It added veracity to Wheeler's work by computer-simulation validation of his formulas. These simulations showed very close agreement between Wheeler's formulas and the simulation results.

For a better understanding of the relationship between the Wheeler and Chu approaches, a Wheeler implementation of a Chu antenna was described. This example helped in the understanding of the significance of the internally stored energy.

For electrically small antennas, Wheeler defined the Q for practical antennas. Wheeler determined the Q for the class of small antennas the occupied volumes of which are cylindrical. He related the occupied volume to the effective volume. The effective volume is fundamentally related to the antenna's Q.

The Wheeler and Chu formulas were used to determine the shape factor that provides minimum Q for cylindrical lumped-element antennas with air cores that are constrained to fit within a given spherical volume. It was determined that the lower bound for the Q of inductor antennas is 4.4 times the Chu lower bound, and that the lower bound for the Q of capacitor antennas is 2.4 times the Chu lower bound.

Wheeler's prediction for the performance of the spherical-cap dipole antenna was also evaluated. It was determined that the lowest Q was achieved when the cap area was slightly less than one-half of the spherical area, and that for this case, the ratio of the total-to-external stored energy was 1.75. This is believed to be the lower bound for electric antennas.

Wheeler's formulas have been used for many years by Wheeler and his alumni for the successful design of electrically small antennas. Wheeler's lumped-element antennas, the capacitor and the inductor, fully account for all stored energy, internal and external: such a simple concept, but yet with such far-reaching applications.

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Introducing the Feature Article Author

Alfred R. Lopez is an IEEE Life Fellow and a BAE Systems (Greenlawn, NY) Hazeltine Fellow and Senior Principal Engineer II. He has almost 50 years of experience in antenna design, propagation analysis, and the design and development of radiating systems. All of this time, he has been with BAE Systems through a heritage linking back to Hazeltine Corporation and Wheeler Laboratories, where he started his career in 1958. Over most of his career, he has specialized in antenna designs and systems for aircraft approach and landing operations. He has also contributed to the fields of electronically scanned array antennas, antennas for cellular communications, and ground reference antennas for differential GPS.

In 1958, he received the BEE from Manhattan College, and in 1963, he received the MSEE from the Polytechnic Institute of Brooklyn. He has published more than 50 papers in IEEE and Institute of Navigation publications. He has been awarded 42 US patents, and has received several awards from the IEEE and BAE Systems. He was the recipient of two Wheeler awards, the IEEE Antennas and Propagation Society Harold A. Wheeler Best Applications Prize Paper Award in 1987, and the IEEE Long Island Section Harold A. Wheeler Award in 1993.

Editor's Comments *Continued from page 8*

Our Other Contributions

In her Education Column, Cynthia Furse reports on the Spring 2006 IEEE AP-S Graduate and Undergraduate Research Awards. There is also a call for applications for the 2007 awards in this issue. In the same column, Gonca Çakir, Mustafa Çakir, and Levent Sevgi present a two-dimensional FDTD simulation tool. It is appropriate for both teaching and applied engineering problems involving indoor and outdoor radiowave propagation, resonators, open and closed periodic structures, planar arrays of antennas, and electromagnetic compatibility problems. Copies of the tool are available free for downloading by readers.

In a very interesting contribution to the Antenna Designer's Notebook, edited by Tom Milligan, H. T. Hui, H. P. Low, T. T. Zhang, and Y. L. Lu introduce a new type of mutual impedance, defined for two receiving antennas excited by a plane wave. They show that the concept can be quite useful for designing adaptive arrays. Although the concept is introduced for a particular type of antenna, it appears to have much broader potential application. A contribution by Jose Ricardo Descardeci is also in the Notebook. It presents a solution to a problem associated with evaluating integrals of rapidly oscillating Bessel functions, one example of which occurs in the Moment Method analysis of rectangular patches.

In the AMTA Corner, edited by Brian Kent, Tommi Laitinen, Sergey Pivnenko, and Olav Breinbjerg present an analysis and simulation of an iterative probe-correction technique used in spherical near-field antenna measurements. Although the technique had previously been used for first-order probes, they are able to show how it can be used for higher-order probes (for example, a dual-ridged horn). They also develop guidelines for the range of applicability for the probe-correction technique.

We have several letters to the Editor in this issue, which are always welcome. One set of letters deals with a broadband ridged horn antenna that was the topic of the AMTA Corner in the April, 2006, issue of the *Magazine*. If you are interested in an antenna with a 100:1 bandwidth, you should read this. Another set of letters concerns a new IEEE standard for safety levels for human RF exposure, which was discussed in the February issue.

If you ever have to give a presentation of any sort (that probably includes everyone reading this), you should read the contribution by Michael Kozak in the PACE column, edited by Michael Johnson. It will improve what you do.

Large arrays of relatively autonomous sensors are finding application in many fields. It is usually important for the positions of the sensors to be known. In the Wireless Corner, edited by Christos Christodoulou and Tuli Herscovici, Aly El-Osery, Wael Abd-Almageed, and Moustafa Youssef present a new algorithm for determining the positions of such sensors. It uses the strength of the signals transmitted from the sensors to mobile nodes equipped with GPS capabilities.

In his AP-S Turnstile column, Rajeev Bansal shares some correspondence he has received relating to two of his previous columns, dealing with whether or not Newton's Third Law of Motion is universally true for charged particles.

Thanks to John Volakis

In 1992 – almost 14 years ago – John Volakis suggested the idea of the EM Programmer's Notebook for this *Magazine*, and agreed to take on the role of soliciting input and editing the column. He has done an outstanding job, bringing us a wealth of interesting and useful information on computational electromagnetics. He recruited David Davidson to join him in editing the Notebook some years ago, and John is now turning the Notebook totally over to David. I deeply appreciate all of the help and wise council John has given me personally over the years with the *Magazine*, and all that he has done for our readers.

Nitpicking

To "nitpick" is to be critical of inconsequential details. The following comes close to this, but what's involved isn't so inconsequential.

I have recently received a number of contributions in which the quantities and units are italicized: e.g., *50 MHz*, *10 dB*, *0.03 S/m*. That simply isn't correct. Almost without exception, quantities and their units should be set in normal type: 50 MHz, 10 dB, 0.03 S/m. I suspect that this started because variables in equations are properly set in italics, in most instances (however, this should also typically be done using, for example, *Equation*

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